Worksheet 3.4: Differentiation Practice

USE THE VALUES IN THE FOLLOWING TABLE TO ANSWER THE QUESTIONS BELOW.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$	$f''(x)$
0	0	1	2	-1	4	-5	0
1	3	2	1	3	-2	-4	-4
2	1	0	3	-2	3	2	1
3	2	3	0	4	2	-3	2

1. Determine if $y = f(x)g(x)$ has a horizontal tangent at $x = 1$.

2. Determine if $y = h(g(x))$ is increasing or decreasing at $x = 3$.

3. Find the equation of the tangent line to $y = f(g(x))$ at $x = 2$.

4. Find $u'(1)$ if $u(x) = \sqrt{h(x) + 3}$

5. Determine if $y(x) = (f(x))^2$ is concave up or down at $x = 1$.

6. Find the slope of $y = \dfrac{g(x)}{x^3}$ at $x = 2$.

7. Find $\frac{dy}{dx}$ for $y = f(g(3))$.

8. Find $u'(4)$ if $u(x) = h(\sqrt{x})$.

9. Find the slope of the tangent line to $y = e^{g(x)}$ at $x = 0$.

Worksheet 3.4: Chain Rule

1. Each of the following functions passes through the origin. As you zoom in at the origin, some of these functions become indistinguishable (i.e., like x^2 and x^3, because both begin to resemble a horizontal line). Group together the functions which look the same near the origin and give the equation of the line that they begin to resemble. (I.e, x^2 and x^3 begin to look like $y = 0$.)

$y = x^2$	$y = x^3$	$y = x^{0.2}$
$y = 1 - \cos x$	$y = \arctan 3x$	$y = \frac{-x}{x+1}$
$y = \sqrt{x}$	$y = 1 - e^x$	$y = \ln(3x + 1)$

2. Assume that $x > 0$ and let $k(z) = \arctan(z) + \arctan(1/z)$.
   (a) Find $\frac{dk}{dz}$.
   (b) Simplify your answer. What does this tell you about k?

3. The radius of a spherical balloon is increasing at a constant rate of 2 cm/sec.
   (a) Write down the relationship between the radius of the balloon and the volume of air it contains.
   (b) What is the volume of air in the balloon when the radius is 10 cm? Include units.
   (c) How fast is the volume changing when the radius is 10 cm? Include units.

Worksheet 3.4: Product, Quotient, and Chain Rule

Part I - Find the indicated derivatives.

1. Find $\frac{d^2y}{dx^2}$ where $y = 2x - \frac{1}{\sqrt[3]{x}} + 3^x - e$.
2. Find $f'(3)$ where $f(z) = \frac{z^2+1}{\sqrt{z}}$.
3. Find $h'(y)$ where $h(y) = (3y^2 + 7y)(2(1.3)^y + 5)$
4. Find $h'(2)$ where $h(x) = 2f(x) \cdot g(x)$ and $f(2) = 7$, $f'(2) = -2$, $g'(2) = -1$, $g'(2) = 3$.

Part II - Given the table below, find the indicated derivatives at $x = 1$ and $x = -2$ where possible.

Table 3.4.1 *function values and derivative values*

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	3	-2	-1
-2	-2	-5	1	7

1. $\frac{d}{dx}[(f(x))^2 - 3g(x^2)]$ at $x = 1$
2. $\frac{d}{dx}[f(x) \cdot g(x)]$ at $x = 1$
3. $\frac{d}{dx}[\frac{f(x)}{g(x)}]$ at $x = -2$
4. $\frac{d}{dx}[f(g(x))]$ at $x = 1$
5. $\frac{d}{dx}[g(f(x))]$ at $x = -2$
6. $\frac{d}{dx}[g(g(x))]$ at $x = -2$
7. $\frac{d}{dx}[f(g(4-6x))]$ at $x = 1$
8. $\frac{d}{dx}[(g(x))^2]$ at $x = 1$

Worksheet 3.7: Derivative Practice and Summary

1. (a) Find the equation of the tangent line to $f(x) = x + \frac{4}{x}$ at the point $(2, 4)$.

   (b) Use your calculator or computer to graph $f(x)$ and the tangent line you found to check your work.

   (c) Do you expect the tangent line approximation to $f(x)$ at $x = 1$ to be an over- or under-estimate? Why?

   (d) Use the tangent line to estimate the value of the function at $x = 1$.

   (e) Compare the actual value of $f(1)$ to your estimate in part (d). Does you result confirm your prediction in part (c)?

2. Find the first derivative of the following functions.

   (a) $y = \dfrac{\tan x}{x}$

   (b) $y = \sin(e^x)$

   (c) $y = e^{\sqrt{x}}$

   (d) $y = \ln(\cos(\theta^2))$

3. Find $\frac{dy}{dx}$ by implicit differentiation: $x^4 + y^4 = 16$.

4. Find an equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(1, 1)$.